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Not one bit of de Sitter information

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Abstract: We formulate the information paradox in de Sitter space in terms of the no-cloning principle of quantum mechanics. We show that energy conservation puts an upper bound on the maximum entropy available to any de Sitter observer. Combined with a general result on the average information in a quantum subsystem, this guarantees that an observer in de Sitter space cannot obtain even a single bit of information from the de Sitter horizon, thereby preventing any observable violations of the quantum no-cloning principle. The result supports the notion of observer complementarity.

Keywords: dS vacua in string theory, Models of Quantum Gravity.

Resolving the black hole information paradox remains one of the most important open problems in theoretical physics [1]. By the principle of equivalence, an in-falling observer crossing a black hole horizon would notice nothing special there. Suppose this observer is carrying a system in some quantum state. If black hole evaporation is a unitary process, the Hawking radiation left after the hole has disappeared must encode a faithful copy of the quantum state that the in-falling observer was carrying. Hence, at earlier times, two copies of the state seem to be needed, one inside (carried by the in-falling observer) and one outside the black hole (encoded in the Hawking radiation). But this seems to fall afoul of the no-cloning principle of quantum mechanics. This principle (which complicates the task of making back-ups in quantum computing) says that no unitary operator can make duplicate copies of a general quantum state; copying violates linearity. Thus even if we were able to show that Hawking radiation encodes information we would still be faced with a serious conflict with quantum mechanics.

A proposed resolution to this dilemma is the principle of black hole complementarity. This in effect permits the cloning to take place so long as no observer can witness it [2, 3]. Although at first sight complementarity may look easy to falsify, a number of careful thought experiments indicate that the principle cannot be ruled out, at least for black holes [4 − 6].

In pure de Sitter space, it is less clear that there is a threat of information loss. Nevertheless, because de Sitter radiation can be derived in much the same way that black hole radiation can [7 − 9], one expects that de Sitter radiation too contains information about things that have fallen through the horizon. We shall assume that to be the case here. Then there are two copies of the quantum state, one carried by the observer who crosses the de Sitter horizon, and one encoded in the de Sitter radiation. But now we again run the risk of observable violations of the no-cloning principle [10]. The purpose of this note is to show that these do not occur: energy conservation protects observer complementarity and no observer observes any duplicate information. In fact, we will derive the stronger result that an observer in de Sitter space cannot retrieve even a single bit of information from semi-classical measurements of the de Sitter radiation.

It helps to first recall a few facts about information. The information contained in a system is the deficit between the maximal coarse-grained entropy that it could have and the entropy that it actually has:

$$I = S_{\text{maximal}} - S_{\text{actual}} \,. \tag{1}$$

Intuitively, the more disordered a system is, the closer it is to thermal equilibrium and maximal entropy, and therefore the less information it contains. A calculation of the information contained in a subsystem of a larger system was done in a remarkable paper by Don Page [11]. Page imagined that the total system was in a pure quantum state within the total Hilbert space, $\mathcal{H}$. Let $\mathcal{H}$ be divided into a tensor product of two Hilbert spaces of dimension m and n, corresponding respectively to the Hilbert spaces of the subsystem and of the rest of the system. Tracing over the states of the rest of the system yields a density matrix, ρ, for the subsystem. Page calculated the entanglement entropy, $-\mathrm{tr}\rho \ln \rho$, for the subsystem, and averaged it over all possible pure states for the total system (where the

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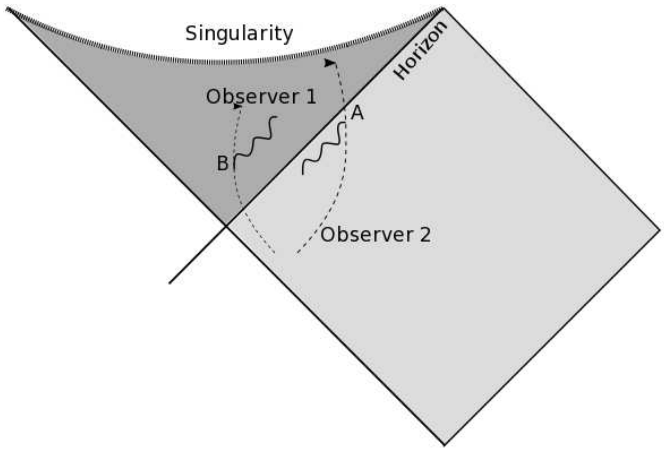

Figure 1: Scenario for potential violation of black hole complementarity.

averaging was with respect to a uniform weighting for pure states given by the unitarily invariant (Haar) measure). The result [11, 12], for $m \leq n$, is

$$\langle S \rangle = \sum_{k=n+1}^{k=mn} \frac{1}{k} - \frac{m-1}{2n} \,. \tag{2}$$

Since $S_{\text{maximal}} = \ln m$, the average information in the subsystem is, by (1) and (2),

$$\langle I_{m,n} \rangle = \ln m - \langle S \rangle \approx \frac{m}{2n} \,, \tag{3}$$

where the approximation assumes that $m, n \gg 1$. When $m = n$, the subsystem and the rest of the system each contain half the entropy. Even so, (3) says that the information contained individually within each of them is typically just half a unit; the bulk of the information is encoded in the correlations between the two parts of the system. Since the information in even a single bit is $\ln 2$, we see that effectively no information is contained in the subsystem. That is, if the total system is in a typical pure state, then a subsystem carries not one bit of information unless it contains at least half the entropy of the total system.

These general considerations have had profound consequences for the black hole information paradox. Consider the situation shown in the Penrose diagram of figure 1. Observer 1 falls into a black hole carrying with him a quantum system of one bit, a spin degree of freedom say. Observer 2 stays outside until, at point A, she has received the Hawking radiation carrying the information about the spin state. She then bravely jumps into the black hole in an attempt to see the same bit twice. To help her do so, observer 1 sends a message to observer 2 containing the spin bit. But observer 1 must send the message no later than point B in order for the bit to reach observer 2 before she hits the singularity. Page's calculation is essential here because it says that only after half the entropy of the hole has been radiated out can any information about the spin state emerge. Hence, in D

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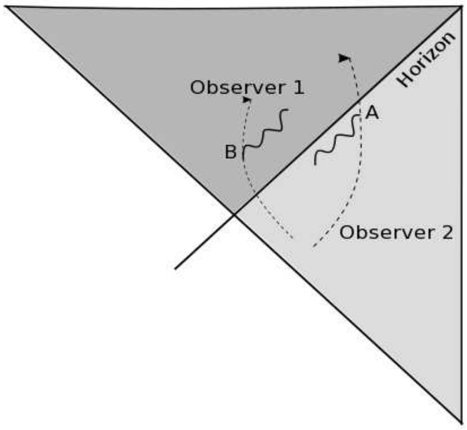

Figure 2: Scenario for potential violation of de Sitter complementarity.

spacetime dimensions, an information retention time of order

$$t_{\text{info}} \sim \left(G_D^2 M^{D-1}\right)^{\frac{1}{D-3}} \tag{4}$$

must elapse before observer 2 can jump into the hole at point A. The delayed in-fall of observer 2 leaves a limited amount of time for observer 1 to send a signal containing the bit; it can then be shown on the basis of the energy-time uncertainty principle that the energy required to send the message would back-react severely on the geometry, thereby invalidating the entire semi-classical set-up [4, 13].

For de Sitter space, the Penrose diagram looks quite similar (see figure 2), but there are a few differences. The first obvious difference is the absence of a singularity. More crucially, the analogue of the information retention time is obscure for de Sitter space. Without a delayed in-fall, the observer who records the Hawking radiation could potentially jump through the horizon and still have enough time to receive the message with the duplicate bit.

Now, in a strong version of holography, the finite entropy of de Sitter space actually enumerates the logarithm of the number of states in the finite Hilbert space $\mathcal{H}$ of de Sitter quantum gravity [14−20]. In D spacetime dimensions, the Bekenstein entropy is $\Omega_{D-2}L^{D-2}/4G_D$ where G_D is Newton's constant, Ω_{D-2} is the volume of a unit D-sphere, and L is the radius of curvature of de Sitter space. We regard the Hilbert space $\mathcal{H}$ of the entire system to have dimension equal to the exponential of this number. Now we would like to estimate how much information is available to an observer inside a de Sitter horizon. As (3) indicates, the information content depends on the entropy of the subsystem available to the observer. In particular, the observer is immersed in a bath of de Sitter radiation. Naively, we might try to estimate its entropy. A gas of blackbody radiation in de Sitter space with $\mathcal{N}$ polarizations, or $\mathcal{N}$ massless degrees of freedom, has entropy

$$S = \frac{4\mathcal{N}D}{(4\pi L)^{D-1}} \frac{\Gamma(D-1)\zeta(D)}{\left(\Gamma\left(\frac{D-1}{2}\right)\right)^2} \left[\int_0^{L-\epsilon} \frac{r^{D-2}dr}{(1-(r/L)^2)^{D/2}}\right] . \tag{5}$$

Here we have used the usual blackbody entropy density at the local temperature and integrated it over the de Sitter horizon volume. This expression diverges if the ultraviolet cut-off ϵ is removed because of the D factors of $\sqrt{1-(r/L)^2}$ in the denominator of the integrand ($D-1$ of which come from the blueshifted temperature with one coming from the proper volume). In fact, even with an ultraviolet cut-off, the entropy in the blackbody radiation can be arbitrarily large if the number of massless degrees of freedom is unrestricted. By this naive estimate then, it appears that the observer could have access to an almost unlimited amount of information.

However, this estimate is no good; we have neglected gravity. Indeed, it is now understood that it is precisely gravitational backreaction, or energy conservation, that allows the quanta to tunnel across the de Sitter horizon [8, 21 − 23] in the first place. But when gravity is taken into account, the maximum entropy configuration consists not of a gas of thermal radiation but of a black hole in de Sitter space. Consider then the line element for Schwarzschild-de Sitter space:

$$\begin{aligned} ds^2 &= -\left(1-\frac{2G_DM}{r^{D-3}}-\frac{r^2}{L^2}\right)dt^2 \\ &\quad+\left(1-\frac{2G_DM}{r^{D-3}}-\frac{r^2}{L^2}\right)^{-1}dr^2+r^2d\Omega^2_{D-2}\,. \end{aligned} \tag{6}$$

For $M < M_{\rm max}$, this has two horizons, a cosmological horizon at radius r_+ and, within it, a black hole horizon of radius r_-. At $M = M_{\rm max}$, the two horizons coincide at the radius r_H; the solution is sometimes called the Nariai black hole. The maximum amount of entropy that can be stored in de Sitter space is clearly the entropy of a Nariai black hole. To find its mass $M_{\rm max}$ we use the fact that the temperature of a de Sitter black hole is

$$T_{\rm hole}=\frac{1}{2\pi}\left(\frac{(D-3)G_DM}{r_-^{D-2}}-\frac{r_-}{L^2}\right)\,. \tag{7}$$

As with extremal black holes in asymptotically flat space, the Nariai black hole has vanishing temperature, a general consequence of coincident horizons. We thus obtain $G_DM_{\rm max}=(D-3)r_H^{D-1}/L^2$. Using $g_{\rm tt}=0$ at the horizon we find

$$G_DM_{\rm max}=\frac{L^{D-3}}{D-1}\left(\frac{D-3}{D-1}\right)^{(D-3)/2} \tag{8}$$

and

$$r_H=\left(\frac{D-3}{D-1}\right)^{1/2}L\,. \tag{9}$$

It follows that

$$\frac{S_{\rm subsystem}}{S_{\rm dS}}\le\left(\frac{D-3}{D-1}\right)^{(D-2)/2}<\frac{1}{e}\,, \tag{10}$$

so that the ratio of entropies is bounded above by e^{-1}, in the limit of infinite D. We see that, for all values of $D > 3$, the entropy in the Nariai black hole is less than half the total entropy of de Sitter space. Thus even if all the Hawking radiation were most efficiently

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captured and stored in a massive black hole, the maximum entropy of the subsystem accessible to the static observer would still be insufficient to encode even a single bit of information.

The spontaneous creation of these massive black holes in de Sitter space violates the second law of thermodynamics because the total horizon area decreases from its maximum value (given by the entropy of empty de Sitter space). Nevertheless, such thermodynamic fluctuations are to be expected, and occur in far less time than a random Poincaré recurrence. This is relatively straightforward to see. Poincaré recurrences occur on a time-scale of the order of the de Sitter entropy $\tau_P \sim e^{S_{\rm dS}} \gg 1$ (in Planck units). In comparison, the probability for Hawking radiating a black hole is $\Gamma \sim e^{\Delta S}$, where $\Delta S = S_f - S_i$ is the change in the entropy of the cosmological horizon before and after emission of the black hole [8]. But since the cosmological and black hole horizons coincide for the Nariai black hole, the entropy of the cosmological horizon after emission equals the Nariai black hole entropy, which was calculated above in (10). Using that result we find (in Planck units and for $D > 3$) that the time-scale τ_N for producing a Nariai black hole is

$$\tau_N \sim \frac{1}{\Gamma} \sim \exp\left[\left(1 - \frac{S_{\rm Nariai}}{S_{\rm dS}}\right) S_{\rm dS}\right] \lesssim \tau_P^{\frac{2}{3}} \ll \tau_P \, . \tag{11}$$

Although the probability of emission is minuscule, the time-scale is nevertheless much smaller than the recurrence time. But what we have shown in this paper is that even such extreme fluctuations do not provide any information for an observer to set up a violation of the no-cloning principle.

We have only considered the scenario of an observer hoping to cross the horizon after recording the Hawking radiation. But it is easy to see that observers who stay on one side or other of the horizon also do not meet with any contradictions. Indeed, all hot horizons can be approximated by Rindler horizons and arguments showing that Rindler observers can be reconciled with complementarity were already made in [4]. To be precise, one assumption that goes into these arguments is that the original pure state is a typical one. In fact it is easy to imagine atypical states for which the above reasoning does not apply. For example, if the total system is in a tensor product of two pure states, then the information contained in the subsystem would be equal to $\ln m$, which could certainly be much more than one bit. But presumably such quantum gravity states would not describe a semi-classical spacetime with a horizon.

In summary, using energy conservation and a basic result in quantum information theory, we have shown that it is impossible for a semi-classical observer in de Sitter space to measure even a single bit of information. No observable violations of the no-cloning principle arise, so observer complementarity in de Sitter space appears to be safe.

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